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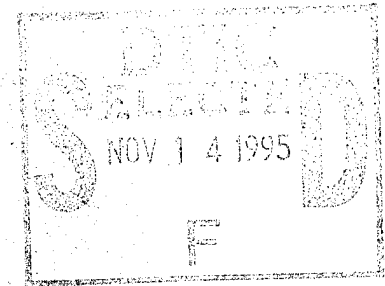
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# BALLISTIC RESEARCH LABORATORIES



REPORT NO. 673

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## A Logical Coding System Applied to the ENIAC

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DTIC QUALITY INSURANCE

ABERDEEN PROVING GROUND, MARYLAND

TECHNICAL INFORMATION BRANCH  
ORDNANCE RESEARCH CORPS  
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## A Logical Coding System Applied to the ENIAC

R. F. CLIPPINGER

"DTIC USERS ONLY"

PROJECT NO. TB3-0007 OF THE RESEARCH AND  
DEVELOPMENT DIVISION, ORDNANCE DEPARTMENT

29 SEPTEMBER 1948

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## A LOGICAL CODING SYSTEM APPLIED TO THE ENIAC

## INTRODUCTION

In the spring of 1947, J. von Neumann suggested to the author that it would be possible to run the ENIAC in a way very different from the way contemplated when it was designed; a way which had very important advantages to be discussed below. Since that time his suggestion has been worked into a finished regime by J. von Neumann, A. Goldstine, B. Bartik, R. Clippinger, and A. Gehring with contributions by A. Galbraith, J. Giese, K. McNulty, J. Holberton, E. Snyder, E. Schlain, K. Jacobi, F. Bilas, and S. Spear. The role of J. von Neumann in working out the details has been a central one.

Sections 1, 2, and 3 make clear the methods by which the sixty orders are effected. However, the reader who wants a more rapid description of the procedure of programming a problem for the ENIAC may skip these sections and read only Sections 4, 5, 6, and 7.

A. It is hoped by the author that this report will make the task of coding problems so clear and straightforward that physicists, aerodynamicists, applied mathematicians, etc. with no prior experience with computing machines can code their own problems and prepare tests. This is of critical importance since the shortage of personnel at Aberdeen will prevent the Ballistic Research Laboratories from being able to code extra-Ballistic Research Laboratory problems in the foreseeable future.

B. Not only will problem coding for the ENIAC be easier but, using the new code in the systematic methods of J. von Neumann and H. Goldstine<sup>1</sup>, it will also be much faster.

C. Problems about four times as long may be coded in the new scheme. A rough measure of this factor is the ratio of the number, 1800, of order positions (see Section 3) in the function tables to the number, 450, of program controls (see Section 1). This gain is achieved at the expense of function table space.

D. Since problems are put on the machine in the new system by setting switches on the function table, problems can probably be changed in an hour instead of a day by the old method where many cables had to be plugged in and out. Furthermore, the switch settings made in the function tables can be systematically and rapidly checked. (See Section 5.)

E. Besides easing the switch settings check, the new method will ease the ENIAC functional testing. On one function table, programs may be set up at will to test any suspected unit. This facility may very well result in causing the ENIAC to deliver results at a greater rate despite the reduction in computing speed by a factor of six. At the time of writing, the ENIAC is being tested about half of the time.

## SECTION I

## GENERAL DESCRIPTION OF THE ENIAC

There exists a five volume report on the ENIAC prepared by A. Goldstine, H. Huskey, and A. Burks for the Moore School of Electrical Engineering. The report cited and section 1 of this report consist of a description of the machine and a method for using it which we shall call local programming. Under this method of programming, the machine is given its instructions for each new problem by the setting of large

<sup>1</sup> H. H. Goldstine and J. von Neumann, "Planning and Coding of Problems for an Electronic Computing Instrument", Institute for Advanced Study, Princeton, N. J. (1947).

stant throughout a computation are stored on the switches of the constant transmitter or of the portable function tables and emitted when needed. The accumulators not only function arithmetically, but also can be used to store numbers which are computed in one part of a computation and required in other parts. All units have program controls which contribute to the programming memory in the following ways:

- (1) by recognizing the reception of a program input signal which stimulates the unit to perform
- (2) by causing the programming circuits to operate (as specified by the setting of program switches when there are options regarding the operation to be performed)
- (3) on the completion of the operation, by emitting a program output signal which, by means of program cable connections to program lines, is brought to other units to cause them to operate. The program cable connections and switch settings are established before the computation begins.

The kind of programming described in points 1, 2, and 3 above is described as local programming memory because it is taken care of locally at each unit for that unit. The master programmer provides a certain amount of centralized programming memory by coordinating the local programming of the other units.

The input devices for the ENIAC consist of the card reader and the constant transmitter mentioned above in connection with numerical memory. The printer and card punch record computed results.

The governing units of the ENIAC are the initiating unit and the cycling unit. The initiating unit has controls for turning the power on and off, starting a computation, initial clearing, and other special functions. The cycling unit converts 100 kc sine waves emitted by its oscillator into a fundamental train of signals repeated every addition time (i.e. repeated 5000 times per second). These signals include various sequences of pulses. The term pulse is used to refer to a voltage change (either positive or negative) from some reference level and the restoration to the reference level which takes place in a short time, between 2 and 5 micro-seconds.

With a few exceptions digits are communicated from one unit of the ENIAC to another in pulse form. Digit trays stacked above the front panels running from accumulator 1 to the second panel of the constant transmitter are used for this transmission. A digit tray has 11 wires and a ground. Each of ten wires carries the pulses for one place of a 10 place decimal number. To represent the digit  $n$  (where  $0 \leq n \leq 9$ ) in a particular decimal place,  $n$  pulses are transmitted over the wire associated with that particular decimal place. The 11th wire is used for the transmission of sign information. No pulses are transmitted for sign plus, and 9 pulses for sign minus (see discussion of complements below). Pulses are transmitted over all 11 conductors simultaneously. Units which are to communicate with one another in the course of a computation have their digit input and/or output terminals connected by means of digit cables to the 12 point terminals on a digit trunk. At any given time, only one 10 digit number with its sign may be transferred over a particular digit trunk. More than 1 unit may listen to this number. Through the use of more than one digit trunk, several different numbers may be transferred simultaneously. The units of the ENIAC transmit numerical information by emitting appropriate numbers of the pulses which they receive from the cycling unit. Addition is performed in accumulators by means of 10 decade counters, one counter for each decimal place of a 10 digit number, and a binary counter for sign plus (P) or minus (M). These counters are advanced one step by each pulse received. The decade counters and PM counter of an ac-

cumulator are so interconnected that provision is made for carry over. Subtraction is performed by adding the negative of the subtrahend to the minuend.

In order to avoid the necessity for cycling counters backwards, the negative of a number is represented as a complement with respect to a power of ten. Let us consider the decimal point to be located at the extreme right of an accumulator. Then the complement with respect to  $10^{10}$  of the positive number

stored in an accumulator as  $P + \sum_{i=0}^9 a_i \cdot 10^i$  is formed by transmitting 9 pulses for sign M and by transmitting the digit pulses for  $10^{10} - \sum_{i=0}^9 a_i \cdot 10^i$ . Similarly, the complement with respect to  $10^{10}$  of the

negative number stored as  $M + \sum_{i=0}^9 b_i \cdot 10^i$  is formed by transmitting no pulses for sign P and by transmitting the digit pulses for  $10^{10} - \sum_{i=0}^9 b_i \cdot 10^i$ .

Because the counters in an accumulator are so connected that there is carry over not only from each decade counter to the one on its left but also from the 10th decade counter to the binary counter for sign, the usual arithmetic properties obtain when complements are used in addition and subtraction.

#### PROGRAMMING BY MEANS OF PULSES, SWITCHES AND CABLES.

Before a computation can be performed on the ENIAC by the old method, not only must the digit input and output terminals of the units be connected into digit trunks for the communication of numerical data, but also the units must be set up so as to recognize when they are to operate and which particular operations are to be performed. Program controls and program trays and cables are used to instruct the ENIAC in the programming requirements for a particular computation.

Each unit of the ENIAC has one or more program controls. These controls are either of the repeat or non-repeat type. Non-repeat program controls have an input terminal for a program signal and a receiver. Repeat controls have both an input and an output terminal for a program signal and a transceiver or some logically equivalent device. Each program control on a unit which is capable of more than one operation or which is capable of performing operations in a variety of ways has a set of program switches.

Receivers and transceivers alike have the following properties: 1) they have two stable states which will be referred to as the normal and abnormal states; 2) when a program input signal is received, they are set into the abnormal state; 3) they are so connected (through the program switches, if any) to the programming circuits that in the abnormal state they cause the programming circuits to function appropriately; and 4) when the required routine has been completed, they are reset to the normal state so that activity in the unit ceases. When the set of instructions either set up on the program switches of a repeat control or built into the programming circuits has been completed, the transceiver of a repeat program control causes a central programming pulse to be emitted as a program output pulse from the program control's output terminal.

The program trays, like the digit trays, contain 11 wires and a ground, and have 12 point terminals at each end, so that as many trays as desired can be connected to form a program trunk. Each of the 11 lines running the length of a program trunk is referred to as a program line. The program trays differ from the digit trays only in that at two foot intervals the program trays have a set of 11 two point program terminals (1 wire and a shield) instead of a 12 point digit terminal. Input and output terminals of program controls are connected to the program lines by means of program cables.

The procedure for instructing the ENIAC in its routine, then, consists, in local programming, of setting program switches on the units so that, when stimulated by a program input pulse, the program controls will cause the units to carry out a set of specific operations. The temporal order in which the operations are to follow one another is determined by the manner in which program pulse input and output terminals are connected to program lines. All program controls whose program pulse input terminals are connected into the same program line start to operate simultaneously when the program line carries a program signal. If one of the program controls thus stimulated is a repeat program control and if its program pulse output terminal is connected to a second program line, all program controls whose program input terminals are connected to this second program line start to operate when the routine set-up on the repeat program control has been completed.

#### **SYNCHRONIZED SYSTEM.**

All units of the ENIAC operate in synchronism with one another, i.e., all units that start to operate at the same time complete their operations either at the same instant or at times that differ by an integral number of addition times. The phrase "complete an operation" covers not only finishing the numerical processes involved in the operation but also the emission of a program output signal.

The basis of this synchronization is the fundamental train of pulses emitted by the cycling unit and delivered to all units of the ENIAC by means of a set of connected trays called the synchronizing trunk. These trays are physically the same as the digit trays. The central programming pulse (CPP) emitted by the cycling unit in pulse time 17 of every addition time cycle plays a major role in such synchronization, since the program output pulse which a repeat program control emits upon the completion of a program results from allowing a CPP to pass. The units of the ENIAC, moreover, have been so designed that in order to complete their operations they require the pulses of either one addition time cycle or of an integral number of addition time cycles.

Even though the electromechanical devices used with the ENIAC, the reader and the card punch, do not take an absolutely definite number of addition time cycles to complete their operations, these units have been integrated into the synchronized system since they have been provided with program controls which emit a CPP as a program output pulse. Units of the ENIAC can even operate in parallel with the card reader since the reader does not emit a program output pulse signifying the completion of reading until it has received as an interlock pulse a program output pulse from some other unit of the ENIAC to indicate that the sequence carried on in parallel with reading has been completed.

Because the units of the ENIAC operate in synchronism with one another and because multiple digit and program trunks have been provided, the operator can schedule parallel operations when planning the set-up of a problem. For example, the multiplier can be operating while several accumulators are performing additions and subtractions and while the divider is finding a quotient. Naturally, the scheduling of parallel

operations requires that the operator plan for the use of separate digit trunks for the various operations and, in some cases, requires that attention be given to the number of addition times needed for the operations.

It is the sacrifice of the possibility of scheduling parallel operations which, surprisingly, leads to the considerable advantages of the system to be described in sections 2-7.

### PROGRAMMING THE ENIAC.

The detailed description of the sequence of operations to be followed by the ENIAC is what we call a set-up table. As formerly carried out it consisted of a set of sheets of paper with about twenty-seven columns, (one for each accumulator and function table, the constant transmitter, the master programmer, the reader, and the printer) on which were listed chronologically downward the program control settings and input and output pulses for each operation. As we briefly describe each unit we shall sometimes show a typical element of a set-up table for that unit.

### THE ACCUMULATOR

The accumulator front panel contains, from bottom to top: program trays and sockets; the program control neons; the input, output sockets; the program controls; digit input and output sockets; digit trays and sockets; and finally the counter neons. Typical parts of it are shown in Figure 1.1. Program pulses are routed from a program line to a program input socket 1i, 2i, etc. by a cable.

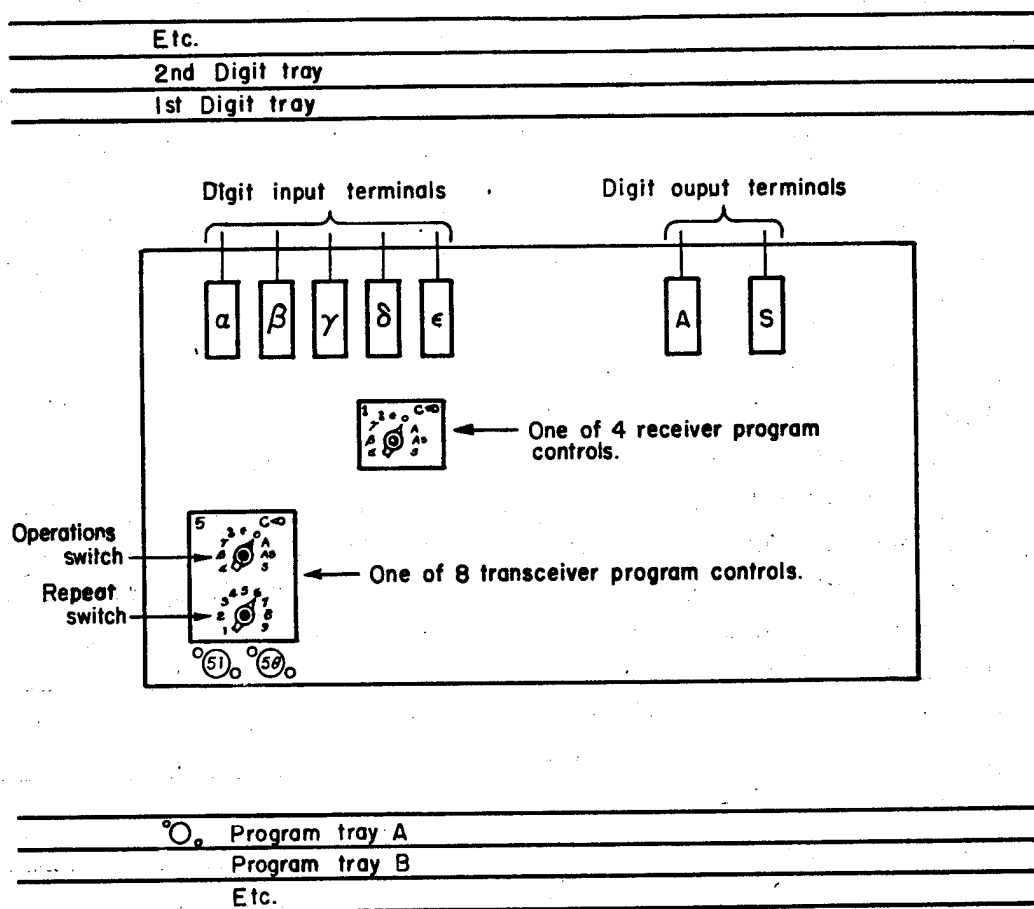


FIG. 1.1



In Figure 1.2, the numbers and associated letters in the top row show how the digit tray sockets and the accumulator input and output sockets are cabled together. For example, in accumulator 7 the  $\delta$  input socket is connected to the second digit tray and the subtract output socket is connected to digit tray 3.

In each row and column the letter and number in the upper left corner indicate the program tray and wire which carry the CPP which stimulates the operation indicated in that box. The next number, if any, indicates with which program input socket the aforementioned wire is connected. The symbols in the center of the box indicate firstly whether the accumulator is to receive through the

$\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$  or  $\epsilon$  input socket, or transmit positively or negatively or both, or do nothing; secondly, whether the accumulator should hold the manner or clear it when finished; and thirdly, how many times the operation should be done. The letter and number at the lower right corner indicate the tray and wire which should carry away the output pulse emitted by the accumulator upon completion of the operation. For example, in Figure 1.2, in the twenty-seventh add-time CP pulse carried by program tray A, wire 3 stimulates accumulators 6, 7, and 8. Accumulator 6 transmits its contents, a, once on digit tray 1 and retains it; accumulator 7 does nothing, then clears; accumulator 8 receives once from digit tray 1 and holds. Thus a, b, and c are replaced by a, 0, and a + c. At the end of add-time 27, accumulator 6 emits a pulse which is carried on program tray A, wire 4, and stimulates accumulator 8 to send -a-c twice to accumulator 7, then clear. This example illustrates set-up tables in general and reminds the reader that addition, subtraction, and transmission are accomplished in one add-time.

Add-time	A-6		A-7		A-8	
	1	A	2	3	1	2
			$\delta$	S	$\alpha$	S
	A-3	(a)	A-3,6	(b)	A-3	(c)
27	A-01		001		$\alpha$ 01	
		A-4	0		(G+C)	
28			A-4 0		A-4 (G+C)	
			$\delta$ 02		9C2	
			(-20-20)		0	A-5

FIG. 1.2

### THE MULTIPLIER

The multiplier has a front panel similar to that of an accumulator and is associated with four accumulators called the multiplier (11), multiplicand (12), left-hand partial product (13), and product accumulators (15). An element of the set-up table for the multiplier is more complicated simply because there are more things to control. The symbols shown in Figure 1.3 indicate that in add-time 56, accumulators 11 and 12 receive into channels  $\alpha$  and  $\beta$  respectively, then in the next 7 + 3 add-times the 7-digit multiplier is multiplied by the 10-digit multiplicand, and in the 12th add-time the answer rounded off to six places is transmitted positively and negatively from accumulator 15 which then clears. The multiplier and multiplicand retain their numbers. At the end of the 11th add-time the CPP is emitted from the multiplier and is carried

Add-time	Multiplier	
56	C-3	
	$\alpha$ 0, $\beta$ 0, 7, 6	
	ASC	
67		C-7
68		

FIG. 1.3

away on wire 7 of tray C to stimulate some accumulator to receive the product. The decimal is at the left, i.e., only the 10 most significant figures of a 20 digit product are retained.

### THE DIVIDER AND SQUARE-ROOTER

The divider is similar to the multiplier, with the two differences that the associated accumulators are 4-quotient, 5-numerator, 7-denominator, 9-shifter, and that division requires an indefinite time averaging approximately 130 add-times. The divider is also the square-rooter but accumulator 4 is not associated with square-rooting and the number appearing in accumulator 7 is twice the square-root. In both cases the number in accumulator 5 is replaced by the remainder.

### THE READER

The reader is a piece of IBM equipment which reads eight ten-digit numbers from a card into relay switches called the constant transmitter. Reading takes about 2500 add-times and like every other operation is stimulated by a CPP and terminates with a CPP.

### THE PRINTER

The eight accumulators 1, 2, 15, 16, 17, 18, 19, and 20 are connected to a piece of IBM equipment called the printer which prints the numbers stored in these accumulators on a card when the printer is stimulated. This requires about 1500 add-times.

### THE CONSTANT TRANSMITTER

Any of the eight numbers from an IBM card sent by the reader to one of the eight positions A, B -- H in the constant transmitter may be sent out on a digit tray in one add-time as a whole, or only the left or right five digits may be sent. For example, the symbols in figure 1.4 indicate that in add-time 1021, the left five digits of the first number are sent out on digit tray one, followed in the next add-time by the right five digits of the second number, and in the next add-time by all ten digits of the fourth number. The numbers transmitted remain in the constant transmitter until a new card is read.

Add-time	A I	B I	D I
1021	D-11 A <sub>L</sub> D-9		
1022		D-9 B <sub>R</sub> E-1	
1023			E-1 D <sub>LR</sub> E-2

FIG. 1.4

### THE FUNCTION TABLES

The 3 function tables each contain a portable bank of switches for 200 six-digit numbers which may be used as 100 twelve-digit numbers and an associated panel of program controls. There is a switch to

govern whether the function table looks up  $F(x-2)$ ,  $F(x-1)$ ,  $F(x)$ ,  $F(x+1)$  or  $F(x+2)$ , where  $x$  is the number from 0 to 99 sent to it in the second add-time, and a switch to determine the number of times the function is transmitted. For example, from figure 1.5, in the 21st add-time a CPP from tray K, wire 2, stimulates function table I. In the next add-time the pulse NC stimulates some accumulator to send an argument from one to ninety-nine to function table I. In the 25th and 26th add-times  $F(\text{arg}-1)$  is transmitted.

FT I	
21	K-2 A-1 NC-2
22	
23	
24	
25	
26	C-5

FIG. 1.5

In the next section we shall see that the function tables provide the memory for orders as well as numbers.

### THE MASTER PROGRAMMER

The master programmer consists of ten 6-stage ring counters called steppers. Each stepper has three inputs and six outputs. The inputs are the direct input, the input, and the clear direct input.  $N$  pulses, CPP or digit, to the direct input immediately advance the stepper  $N$  stages. A CPP sent to the input causes a CPP to be emitted one add-time later from the output socket associated with that stage of the stepper in which it finds itself; a pulse to the clear direct input immediately clears the stepper to the first stage. The master programmer was designed to permit repeating a sequence of operations an arbitrary fixed number of times and then do another sequence of operations. However, the conditional transfer order to be discussed in sections 3, 4, 5, 6, and 7 has obviated this use of it and replaced it with another which we shall discuss in the next section.

## SECTION II

### CONVERSION OF DIGIT PULSES TO PROGRAMMING PULSES

Reduced to simplest language the basic notion of von Neumann's programming method is to build complicated routines from a small (not necessarily minimal) set of elementary orders. In our case, the number of these elementary orders is sixty. Von Neumann's largest single contribution to the application of his methods to the ENIAC was to point out how 60 distinct two-digit numbers can be converted into 60 distinct CPP's, each of which can in turn stimulate one of the sixty desired orders.

It would have been possible to use a vocabulary of 51 orders without changing the ENIAC at all. However, by the addition of four small units, it became possible to speed the process up considerably and have 60 orders; therefore, these units were constructed. The first of them is a ten-stage stepper which, associated with the ten six-stage steppers of the master programmer, constitute the converter. This device operates as follows: one digit,  $\alpha$ , of a two-digit number,  $10\alpha + \beta$ , is fed into the direct input of the ten-stage stepper, setting it to stage  $\alpha$ ; the other digit,  $\beta$ , is fed to the direct inputs of the ten six-stage steppers

setting them to stage  $\beta$ . A CPP to the ordinary input of the ten-stage stepper causes a CPP to go from the  $\alpha$  th output of that stepper to the ordinary input of the  $\alpha$  th stepper. One add-time later, a pulse  $\alpha\beta$  is emitted from stage  $\beta$  of stepper  $\alpha$ .

Since  $\alpha$  can be 0, 1, ---, 9 and  $\beta$  can be 0, 1, ---, 5, there are sixty distinct pulses which can be obtained by converting the corresponding two-digit number. As a matter of practice, the pair of digits  $\alpha\beta$  comes from the function table, but not directly. Instead, a twelve-digit number is sent six times from the function table to a device we shall call the order selector. It is a set of twelve gates controlled in pairs by a six-stage ring. The order selector allows the first pair of digits to pass the first time, the second pair the second time, etc., until all six pairs of digits from one row of the function table have been converted, at which time a pulse from the sixth stage of the selector ring causes the function table argument to be increased by one and clears the selector ring back to stage one.

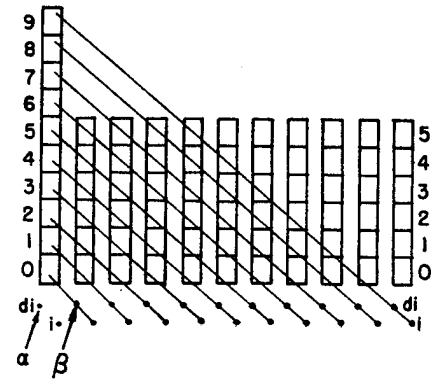


FIG. 2.1

There are three function tables and we may desire to use any argument with any one of the three function tables. This is done by putting a three-digit address in accumulator 6 in the right three digit positions (called positions 1, 2, 3). The third digit,  $i$ , is first sent to a six-stage stepper called the function table selector which emits a pulse one add-time later to stimulate the corresponding function table  $F_i$ . The function table itself then stimulates accumulator six to transmit the first two digits  $10\delta + \gamma$  for an argument.

ORDER BASIC SEQUENCE		F.T. SEL.	ORDER SEL.	10 STAGE STEPPER	MASTER PROG.		ACC. 6		ACC. 20		F.T. I	F.T. II	F.T. III	SC
	C-5	IS	DUAL OF	D-6.	C-5	BEGINS	BASIC	SEQ.			NC		J-4	
	D-6 is DUAL OF E-1	E-11 #1 E-8 di E-91 #2 1 → E-11 2 → F-1 3 → F-2				D-6 is DUAL OF J-2	J-2 A01 E-9  J-4 A01 F-3 F-4 eC1  F-5 002 G-8		C-5 006  E-10		E-11 A-2 NC1	F-1 A-2 NC1	F-2 A-2 NC1	F-3 001 F-5
			F-3 CPP1 F-4  G-8	F-5 cd1  E-10	F-5 cd1									

Figure 2.2

It then transmits  $F_i (10\delta + \gamma - 2)$  to the order selector whence a certain pair of digits,  $10\beta + \alpha$ , of  $F_i (10\delta + \gamma - 2)$  goes to the convertor to be converted to a pulse to stimulate the corresponding operation. This whole basic sequence is summarized in detail in Figure 2.2. The basic sequence precedes every order; however, in the case of most orders it is programmed to start before the preceding order has finished. Thus, though the basic sequence takes seven add-times, the basic sequence plus any one of many of the orders also takes seven add-times.

The pulses which stimulate the sixty orders are listed in the following table.

STEPPER		A	B	C	D	E	F	G	H	J	K
STAGE											
5		S-5	L-11	H-5	S-10	L-4	E-3	H-10	G-2	C-10	V-7
		05	15	25	35	45	55	65	75	85	95
4		S-4	L-10	H-4	S-9	L-3	H-7	H-8	V-1 or E-7	D-11	V-6
		04	14	24	34	44	54	64	74	84	94
3		S-3	L-9	H-3	S-8	L-2	H-6	V-3	E-4 or E-6	D-10	V-8 or V-4
		03	13	23	33	43	53	63	73	83	93
2		S-2	L-8	H-2	S-7	L-1	C-9	O-2	H-11	D-9	H-0 or V-2
		02	12	22	32	42	52	62	72	82	92
1		S-1	L-7	H-1	V-9	C-2	L-6	O-1		D-8	C-8 or C-3
		01	11	21	31	41	51	61	71	81	91
0		C-1	S-6	C-7	E-5	S-11	L-5	B-1	V-5	D-7	C-11
		00	10	20	30	40	50	60	70	80	90

Table 2.I

### THE PULSE AMPLIFIERS

Often it is desirable to have many different pulses stimulate the same operation (as well as distinct ones). This may be accomplished with the pulse amplifiers (eight units, 11 amplifiers per unit) using the one-way pulse transmission property of a vacuum tube. In the following table is shown the hierarchy of CPP pulses obtained with the pulse amplifiers.

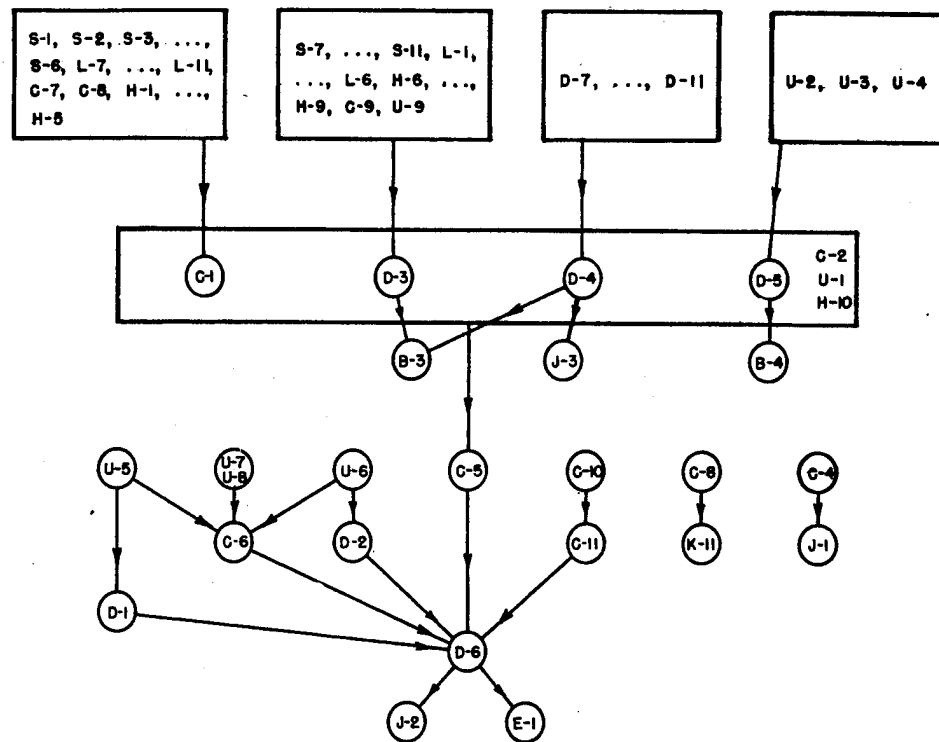


TABLE 2. II

## SECTION III

## DETAILED DESCRIPTION OF THE SIXTY ORDERS

We have seen that there are five kinds of storage places used in the ENIAC. Numbers may be moved between these places but not arbitrarily. From a card in the reader numbers may only be sent to the constant transmitter; from the constant transmitter numbers may only be sent to one of the twenty accumulators; likewise numbers in the function tables can only go to the accumulators; numbers in accumulators may be sent to other accumulators or from certain special accumulators to the printer; from the printer the numbers may only be taken by carrying away the IBM cards. It is accordingly clear that a considerable number of the 60 available orders must provide for moving numbers about. In fact there are 50 orders used for this purpose. Before describing them we should remark that actually 67 orders are set up on the ENIAC making two sets of orders possible; 53 are common to both sets. Of the dual orders we shall name those of the first set 73, 74, 91, 92, 93, 94, 95 and those of the alternate set 73a, 74a, 91a, 92a, 93a, 94a, 95a.

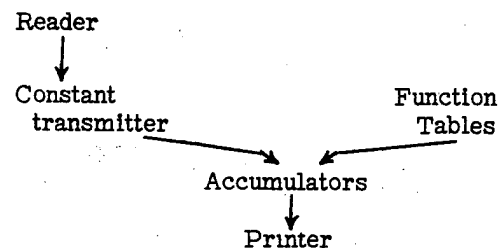


Figure 3.1

**READER TO CONSTANT TRANSMITTER**

The order which sends the eight ten-digit numbers from the reader to the constant transmitter we call the read order, order 62 (code name: Rd). It requires a little over a half second to complete.

**CONSTANT TRANSMITTER TO ACCUMULATORS 11 AND 15**

Since there are ten ten-digit numbers in the constant transmitter (In addition to the eight coming from the reader, there are two fixed ten-digit numbers set with switches.) and twenty accumulators, two hundred orders would be required to make it possible to send any number immediately to any accumulator. It would be possible to have one order which caused all ten numbers to go to a specific set of ten accumulators. However, this would mean that all ten accumulators had to be clear before the order could be given. The compromise agreed upon for these two sets of 60 orders is a set of five "constant transmitter" orders, 80-84 (code names: AB, CD, EF, GH, JK), each of which sends two numbers, A&B, C&D, E&F, G&H, or J&K to accumulators 11 and 15 respectively. J&K store the two ten-digit arbitrary constants.

Consider order 80 for example: From table 2.I we see that order 80 starts with pulse D-7, which via the pulse amplifiers blossoms into pulses D-4, B-3, C-5, J-3, D-6, J-2, and E-1.

ORDER														
80 - C.T.		F.T. SEL.		ACC. 6		ACC. 11		ACC. 15		ACC. 20	F.T. I	F.T. II	F.T. III	C.T.
(A <sub>LR</sub> , B <sub>LR</sub> ) G-5 IS DUAL OF D-6														
D-7 IS DUAL OF D-4 D-4 IS DUAL OF C-5, J-3, & B-3														
	D-6 is DUAL OF E-1	E-1 i #1 E-8 di E-9 i #2 1 → E-11 2 → F-1 3 → F-2	D-6 is DUAL OF J-2	J-2 A01 E-9		J-3 OC1 <sub>T-6</sub> T-6 ø 01		B-3 ø 01						D-7 B <sub>LR</sub> * 1 * 1 A <sub>LR</sub>

Figure 3.2

From Figure 3.2 we see that pulse D-7 causes the constant transmitter to send out B, then A on digit tray one (see digit tray hook-up). B-3 causes accumulator 15 to receive B from digit tray one, adding it to its previous contents; J-3 causes accumulator 11 to clear, then receive A from digit tray one. C-5 causes the basic sequence for the next order to start while A and B are moving from the constant transmitter to accumulators 11 and 15. This is a 7 add-time order.

**ACCUMULATOR TO ACCUMULATOR**

There are, in the first and alternate sets of 60 orders, 39 and 38 orders respectively which move numbers from one accumulator to another. We shall discuss first the 34 orders in this category common to both sets.

There are sixteen orders 01, 02, 03, 04, 05, 10, 11, 12, 13, 14, 15, 21, 22, 23, 24, 25, controlling accumulators 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 15, 17, 18, 19 respectively which we may name "accumulator X listen", "X<sub>L</sub>", orders. Of these, order 01 is typical. Order 01 starts (see tables 2.I and 2.II) with S-1, which leads to C-1, C-5, D-6, J-2, and E-1. S-1 (see figure 3.3) causes accumulator 1 to clear and then

receive from digit tray two; C-1 stimulates a dummy in the constant transmitter whose only purpose is to delay J-1 which causes accumulator 15 to transmit positively onto digit tray two then clear; C-5, of course, starts the basic sequence for the next order. The effect, then, of order  $X_1$  is for accumulator X to clear, then receive from accumulator 15 which then clears.

ORDER

			ACC. 1					ACC. 15				CT
01 - ACC. 1 <sub>1</sub>		S-1 is DUAL OF C-1	S-1 AC1 0-8 0-8 α01					J-1 AC1			C-1 is DUAL OF C-5	C-1 001 J-1

Figure 3.3

Order 20 (code name: 13<sub>1</sub>) differs from these orders in that accumulator 13 does not clear before receiving from accumulator 15.

There are likewise 16 orders, 31, 32, 33, 34, 35, 40, 41, 42, 43, 44, 45, 51, 52, 53, 54, 64 controlling accumulators 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 15, 17, 18, 19 respectively, which we may name "accumulator X talk" or " $X_t$ " orders. The effect of  $X_t$  is for accumulator X to send its number positively to accumulator 15 with both accumulators holding. This leads to addition in accumulator 15.

Order 50 (code name: 13<sub>t</sub>) is similar except that accumulator 13 clears after sending.

In the alternate set of orders, orders 91a, 92a, 73a and 74a (code names: 20<sub>1</sub>, 20<sub>t</sub>, 6<sub>1</sub>, 6<sub>t</sub>) are used for accumulator 20 listen, accumulator 20 talk, accumulator 6 listen, and accumulator 6 talk respectively.

In the first set of orders, accumulator 6 is treated essentially like four smaller accumulators: respectively digits 1, 2, 3; digits 4, 5, 6; digits 7, 8; and digits 9, 10, 11. Order 92 (code name: 6 (11, 10, 9)) adds the numbers in accumulator 6 (11, 10, 9) and accumulator 15 (11, 2, 1) and leaves the sum in both. Order 93 (code name: 6 (8, 7)) adds the numbers in accumulator 6 (8, 7) and accumulator 15 (2, 1), leaving the sums in accumulators 6 (8, 7) and 15 (2, 1). All these accumulator orders take 7 add-times. To control accumulator 20, order 91 (code name: 18 ↔ 20) is provided which takes 9 add-times and interchanges the numbers in accumulators 18 and 20 via accumulator 15.

#### THE TRANSFER ORDERS IN THE FIRST SET

Any order which sends a number to accumulator 6 (3, 2, 1) we shall call (with J. von Neumann) a transfer order because the next order will automatically be taken from a different part of the function table, i.e. the control is transferred from one address to another. Order 73 (code name: 6R3) is the simple transfer for the first set. It causes accumulator 6 (3, 2, 1) to clear and receive the number in accumulator 15 (3, 2, 1) and then clears accumulator 15. We might call order 74 (code name: 6R6) the transfer with a rider. It causes accumulator 6 (6, 5, 4, 3, 2, 1) to clear, then receive the number in accumulator 15 (6, 5, 4, 3, 2, 1) and then clears accumulator 15. The purpose of this order is to make it possible to store an extra address in accumulator 6 (6, 5, 4). Both of these transfer orders take 13 add-times.



### FUNCTION TABLE TO ACCUMULATOR ORDERS

The first system has four function-table-to-accumulator orders and the alternate system has five, of which two are transfers. Two of these orders are common to both sets. The first is order 72 (code name: FT). Accumulator 11 is cleared, then the function table transmits 12 digits and 2 signs as F (address stored in accumulator 8 (3, 2, 1)). Accumulator 11 receives the left sign and the left 6 digits at far left, and accumulator 15 receives the right sign and the right 6 digits at far left. The argument in accumulator 8 is then increased by one. The time consumed is 13 add-times.

Order 70 (code name: N2D) we call the next two digits order. It operates to send accumulator 15 (2, 1) the two digits following the order 70 in the function table. Fourteen add-times are required.

In the first system the orders 94 and 95 (code names: N4D, N6D) operate similarly to send the next four and next six digits to accumulator 15 in 20 and 26 add-times respectively.

The alternate system contains an order 93a (code name: N3D8) which sends, in 20 add-times, an address from the next two order positions in the function table to accumulator 8 (3, 2, 1) to be used as address for order 72. The first digit in the next two order positions is added to the digit in accumulator 8 (4); accumulator 8 (3, 2, 1) is cleared before receiving.

In the alternate system the transfers 94a and 95a (code names: N3D6, N6D6) send in 20 or 26 add-times one or two addresses from the next two or three order positions in the function table to accumulator 6. In order 94a, as in 93a, the first digit in the next two order positions is added to accumulator 6 (4).

### ACCUMULATORS TO PRINTER

The last transmission order, order 61 (code name: Pr), causes the next IBM card to move into position in the printer and then the numbers in accumulators 1, 2, 15-20 to be printed on the card and held in the accumulators. One second is required.

### THE TEN OPERATIONS

It is not quite true that each of the fifty transmission orders we have just described accomplishes transmission only. All the " $X_t$ " orders, for example, are also addition orders; the transfer orders perform the important operation of shifting the control. The following ten orders, however, are more explicitly operative.

Order C. Order 00 (code name: C) simply clears accumulator 15 and takes 7 add-times.

Halt. Order 71 stops the ENIAC. It is the only order which does not stimulate the basic sequence for the next order.

M. Order 63 replaces the number in accumulator 15 by its complement via accumulator 13 in 7 add-times; for example, to form  $a-b$ , send  $b$  to accumulator 15, use order 63, then send  $a$  to accumulator 15.

D.S. Order 65 replaces the plus or minus sign in accumulator 15 by the plus sign. This is not the same as the change sign order 63 even if the number is negative since it does not affect the first ten digits. This order is added to facilitate the use of accumulators to hold two numbers: an arbitrary number and a number of known sign. This order takes 7 add-times.

X. Order 30 is the multiplication order. To use it, the multiplier must have been previously put in accumulator 11, the multiplicand in accumulator 15, and any number to be added to the product in accumulator 13. After multiplication the product plus the contents of accumulator 13 are placed in ac-

cumulator 15 and the multiplier and multiplicand remain in accumulators 11 and 12; accumulator 13 clears. This order takes 15 add-times. The factors are treated as if they had ten digits each. Only ten places of the partial products are kept. It follows that the answer may be incorrect in the 10th place from the left, by 14 for maximum error or 7 for the average error.

÷ Order 55 causes the number in accumulator 15 to be divided by the number in accumulator 7. During this process the number in accumulator 5 is replaced by the remainder and the quotient goes to accumulator 15. Accumulator 13 must be previously cleared. Assuming that the decimal point in the numerator and denominator is at the extreme left, the quotient has its decimal point two places to the right, and therefore the numerator must not exceed the denominator by more than a factor of 100. Order 55a is arranged so that the quotient is not shifted and therefore the numerator must not exceed the denominator. The time required for division is variable but is of the order of 130 add-times.

✓ Order 60 differs from order 55 only in that the operation is different and accumulator 4 is not involved formally. The radicand must not exceed .2499999999.

Sh and Sh'. Orders 90 and 85 are called shift and shift' respectively. Each takes two order positions in the function tables and requires 20 add-times to complete. The shift order causes the number in accumulator 15 to be shifted as indicated by the two digits in the next order position. The first of these two digits is 0 or 9 according as the shift is to the left or right and the second digit defines the magnitude of the shift, from 1 ---5 inclusive. In the shift' order 85, the number in accumulator 15 is shifted as in order 90, but also the complementary amount in the complementary direction and put in accumulator 12. Thus 85 04 would replace a in accumulator 15 by  $10^4a$  and put  $10^{-6}a$  in accumulator 12.

C.T. The most important order, from the point-of-view of control, is order 75, the conditional transfer. The effect of the order is to shift the control from the address in accumulator 6 (3, 2, 1) to accumulator 6 (6, 5, 4) if the number in accumulator 15 is positive or zero. If the number is negative, the program continues with address in accumulator 6 (3, 2, 1). Sensing the sign of the number in accumulator 15 is accomplished by sending the pulses from the P-M line of the add output socket of accumulator 15 to a two-stage stepper called P-M discriminator no. 2. Shifting the control is accomplished by sending the number from accumulator 6 to accumulator 10 through a shifter which shifts 6, 5, 4 to 3, 2, 1, clears 3, 2, 1, and leaves the other digits alone; the number is then returned to accumulator 6. In section 6 we shall consider more carefully the use of this order. It is immediately clear that because of it the machine can make any choice of sequences of operations that can be made to depend on the sign of a number. Most computations involve more or less simple inductions which in turn depend on decisions based on the sign of a number.

#### SECTION IV

##### SUMMARY OF ORDERS

A summary of orders is found on the following four pages.

## ORDERS FOR 60 WORD VOCABULARY - November 13, 1947

Code Number	Code Name	Order	Contents of Affected Accs.		Add. Times
			Before	After	
00	C	Clear Acc. 15	(15)a	(15)0	7
01,02,03 04,05,10 11,12,13 14,15, 21-25	$x_1$	Acc. x clear, and then receive from acc. 15 which transmits and then clears ( $x \neq 6,13,15,20$ )	(x)a (15)b	b 0	7
20	$13_1$	Acc. 13 hold and receive from acc. 15, which transmits, then clears	(13)a (15)b	a + b 0	7
31-35, 40-45, 51-54,64	$x_t$	Acc. x transmit-hold to acc. 15 ( $x \neq 6,13,15,20$ )	(x)a (15)b	a a + b	7
50	$13_t$	Acc. 13 transmit-clear to acc. 15	(13)a (15)b	0 a + b	7
30	X	No round-off, 10 place multiplier. The number, a in acc. 11 is multiplied by the number d, in acc. 15, added to the number c in acc. 13, and stored in acc. 15. At the end acc.'s 11,12,13 hold a, 0, and d respectively.	(11)a (12)b (13)c (15)d	a d 0 ad + c	15
55	$\div$	Divide and round-off to 10 places. The number c in acc. 15 is divided by the number b in acc. 7, and the quotient is stored in acc. 15, the remainder in acc. 5. Acc. 13 must be clear at the beginning. The alternate division order uses a special + 2 shifter at 4 $\alpha$ that takes data arriving on line 9 and puts it into counter 11. For this reason, denominator must always exceed numerator and, if decimal points of numerator and denominator are at extreme left, decimal point of quotient is also at extreme left.	First digit place at left of b must be zero. (5)a (7)b (13)0 (15)c	rem. b 0 c/b	approx. 75
60	$\sqrt{\phantom{x}}$	Round-off to 10 places. The square root of the number c, in acc. 15 is put into acc. 15 and the remainder in acc. 5. Acc. 13 must be clear at the beginning. A +1 shift of $\sqrt{\phantom{x}}$ is made by sending twice square root to acc. 15 five times. This means that if decimal point of radicand is at extreme left, decimal point of $\sqrt{\phantom{x}}$ is also at far left.	(5)a (13)0 (15)c	rem. 0 c	approx. 75
63	M	(complement of no. in acc. 15)	(13)0 (15)a	0 -a	7
65	DS	Drop the sign of the number in acc. 15. Note that if number in acc. 15 is originally -a, it will become ( $10^{10}$ -a); if it is originally, a, it will remain, a.	(13)0 (PM counter of 15) $\pm$ (counters 1-10) a	0 + a	7
90	Sh	Shift no. in acc. 15 as described below. Shift order takes two positions: 1st position: code symbol 90 2nd position: $10j + k$ where j indicates direction (0 to left, 9 to right) k indicates number of places ( $1 \leq k \leq 5$ )	(13)0 (15)a	$10^k \cdot a$ if j=0 (13)0 (15) $10^{-k} \cdot a$ if j=9	20

## ORDERS FOR 60 WORD VOCABULARY - November 13, 1947 (Continued)

Code Number	Code Name	Order	Contents of Affected Accs. Before	After	Add. Times
71	Halt	Do not get next order (all other orders include getting next order) and therefore cease computing.			
80	AB	Clear acc. 11, then constant transmitter send	(11)x	a	
81	CD	1st. no. mentioned to acc. 11 and 2nd no. mentioned to acc. 15.	(15)y	y + b	7
82	EF		Const. Trans		
83	GH		(A)a	a	
84	JK		(B)b	b	
70	N2D	Send next pair of instruction digits to acc. 15. This order occupies two order positions. 1st position: code symbol 70; 2nd: $10\alpha + \beta$ where $0 < \alpha < 9, 0 \leq \beta \leq 9$	(13)0 (15)x	(13)0 (15)x + $10\alpha + \beta$	14
72	F.T.	Clear acc. 11. F.T. transmit 12 digits and 2 signs from F.T. and line specified in acc. 8 (3,2,1). Acc. 11 receive left sign and 6 digits. Acc. 15 receive right sign and 6 digits. Increase arg. in acc. 8 by 1. Nos. are received at far left of 11 and 15.	(8) places 3,2,1 a (11)x (15)y	a + 1 L(a) R(a)	13
61	Pr.	Move next card into printing position and print contents of accs. 1, 2, 15-20. Move next card into read position.	Move and print		60 cards per min.
62	Rd.	Store information from card on relays of ENIAC reader.			100 cards per min.
75	C.T.	Examine no., a, in acc. 15 and then clear acc. 15. 1) If number in acc. 15 is negative, continue with next order on line specified at 6(3,2,1) 2) If positive, transfer 6(6,5,4) $\rightarrow$ 6(3,2,1) and continue with FIRST ORDER of line now specified in 6(3,2,1)	Acc. 6 11-7 x 6-4 a 3-1 b Acc. 13 0 Acc. 15 a	If a $\geq 0$ Acc. 6 11-7 x 6-4 0 3-1 a Acc. 13 0 Acc. 15 0  If a < 0 Acc. 6 11-7 x 6-4 a 3-1 b	14      8
85	Sh'	Shift no. located in acc. 15 as described below. Shift' order takes 2 positions: 1st position: code symbol 85 2nd position: $10j + k$ The symbols j and k are interpreted in the following way: The no. which is in acc. 15 at the start of this order is shifted as described in the ordinary shift order (code symbol 90). But, furthermore, the no. which is in acc. 15 at the start of this order is also operated on by a shift of $100 - (10j + k)$ , in the sense of the ordinary shift order. The no. resulting from this latter shift is placed in acc. 12.	(12)a (13)0 (15)b  (12) a (13) 0 (15) b	If j=0    If j=9    $10^{-(10-k)} \cdot b$ 0 $10^k \cdot b$  $10^{(10-k)} \cdot b$ 0 $10^{-k} \cdot b$	       20

N.B. The sign of the no. stored in acc. 15 before this order is given is preserved with the nos. that result in accs. 12 and 15 from this order.

## ORDERS FOR 60 WORD VOCABULARY - November 13, 1947 (Continued)

Code Number	Code Name	Order	Contents of Affected Accs. Before	After	Add. Times
93a	N3D8	Clear acc. 8(3,2,1). Send next 3 instruction digits to acc. 8(3,2,1). This order consumes 3 order positions. 1) 1st position: code symbol 93 2) 2nd position: $10\alpha + \beta$  In general, it is assumed $\alpha = 0$ . But if $\alpha \neq 0$ , $\alpha$ is added to whatever decade 4 of the acc. 8 holds before this order is given. $\beta \rightarrow \text{acc. 8(3)}$ 3) 3rd position: 10 $\gamma \rightarrow 8(2)$ $\delta \rightarrow 8(1)$	Acc. 8 11-4 x 3-1 y Acc. 13 0	Acc. 8 $11-4x + \alpha$ $3-1 10^2\beta + 10\gamma + \delta$ Acc. 13 0	20
94a	N3D6	Clear acc. 6(3,2,1). Then send next 3 instruction digits to acc. 6(3,2,1). Take next order from FIRST ORDER POSITION of line now indicated in acc. 6(3,2,1). This order consumes 3 order positions. 1) 1st order position: code symbol 94 2) Remarks made above in N3D8 apply here if acc. 6 is substituted wherever acc. 8 is mentioned in N3D8.	Acc. 6 11-4 x 3-1 y Acc. 13 0	Acc. 6 $11-4x + \alpha$ $3-1 10^2\beta + 10\gamma + \delta$ Acc 13 0 Next order is taken from F. T. 20 $\beta + 1$ ( $\beta = 0, 1, 2$ , for Table 1,2,3, resp.) 1st pos. of line ( $10\gamma + \delta$ )	
95a	N6D6	Clear acc. 6(6-1). Then send next 6 instruction digits to acc. 6(6-1). Take next order from FIRST ORDER POSITION of line now indicated in acc. 6(3,2,1). This order consumes 4 order positions. 1) 1st position: code symbol 95 2) 2nd position: $10\alpha + \beta$ $\alpha \rightarrow 6(6), \beta \rightarrow 6(5)$ 3) 3rd position: $10\gamma + \delta$ $\gamma \rightarrow 6(4), \delta \rightarrow 6(3)$ 4) 4th position: $10\epsilon + \rho$ $\epsilon \rightarrow 6(2), \rho \rightarrow 6(1)$	Acc. 6 11-7 x 6-4 y 3-1 z Acc. 13 0	Acc. 6 11-7 x $6-4 10^2\alpha + 10\beta + \gamma$ $3-1 10^2\delta + 10\epsilon + \rho$ Acc. 13 0	26
91a	$20_1$	Same as $x_1$ (see common orders)	(20) a (15) b	b 0	7
92a	$20_t$	Same as $x_t$ (see common orders)	(20) a (15) b	a a + b	7
73a	$6_1$	Same as $x_1$ (see common orders) BUT take next order from FIRST ORDER POSITION of line specified at 6(3,2,1)	(6) a (15) b	b 0	9
74a	$6_t$	Same as $x_t$ (see common orders).	(6) a (15) b	a a + b	7

## ORDERS FOR 60 WORD VOCABULARY - November 13, 1947 (Continued)

Code Number	Code Name	Order	Contents of Affected Accs. Before	After	Add. Times
94	N4D	Send next 4 instruction digits to acc. 15. This order takes 3 order positions. 1) 1st position: code symbol 94 2) 2nd position: $10\alpha + \beta$ where $\alpha \rightarrow 15(4)$ $\beta \rightarrow 15(3)$ 3) 3rd position: $10\gamma + \delta$ where $\gamma \rightarrow 15(2)$ $\delta \rightarrow 15(1)$	(13) 0 (15) a	(13) 0 (15) $a + 10^3\alpha$ $+ 10^2\beta$ $+ 10\gamma + \delta$	20
95	N6D	Send next 6 instruction digits to acc. 15. This order takes 4 instruction positions. 1) 1st position: code symbol 94 2) 2nd position: $10\alpha + \beta$ where $\alpha \rightarrow 15(6)$ , $\beta \rightarrow 15(5)$ 3) 3rd position: $10\gamma + \delta$ where $\gamma \rightarrow 15(4)$ , $\delta \rightarrow 15(3)$ 4) 4th position: $10\epsilon + \zeta$ where $\epsilon \rightarrow 15(2)$ , $\zeta \rightarrow 15(1)$	(13) 0 (15) a	0 $a + 10^5\alpha + 10^4\beta$ $+ 10^3\gamma + 10^2\delta$ $+ 10\epsilon + \zeta$	26
91	18 $\leftrightarrow$ 20	Accs. 18 and 20 interchange contents	(15) 0 (18) a (20) b	(15) 0 (18) b (20) a	9
92	6(11,10,9)	Acc. 15(11,2,1) transmit-clear to Acc. 6(11, 10,9). Then Acc. 6(11,10,9) transmit-hold to Acc. 1(11,2,1). Then Acc. 1(11,2,1) transmit-clear to 15(11,2,1).	Acc. 6 11-9 x 8-1 y Acc. 13 0 Acc. 15 10-3 0 11,2,1 a	x + a y 0 0 x + 2	7
93	6(8,7)	Acc. 15(2,1) transmit-clear to Acc. 6(8,7). Acc. 6(8,7) transmit-hold to Acc. 1(2,1). Then, Acc. 1(2,1) transmit-clear to Acc. 15(2,1).	Acc. 6 11-9 x 8,7 y 6-1 z Acc. 13 0 Acc. 15 11-3 0 1,2 a	x y + a z 0 0 y + a	7
73	6R3	Clear 6(3,2,1) and then 15(3,2,1) transmit-clear to 6(3,2,1). Get next instruction from FIRST ORDER POSITION of line now specified 6(3,2,1)	Acc. 6 11-4 x 3-1 y Acc. 13 0 Acc. 15 3-1 b	x b 0 0 0	13
74	6R6	Clear 6(6-1) and then 15(6-1) transmit-clear to 6(6-1). Get next instruction from FIRST ORDER POSITION of line now specified at 6(3,2,1).	Acc. 6 11-7 x 6-4 y 3-1 z Acc. 13 0 Acc. 15 6-4 a 3-1 b	x a b 0 0 0 0	13

## SECTION V

## A SUGGESTED TEST PROGRAM

The ENIAC is complicated. It has about 20,000 tubes and thousands of switches and plug-in contacts. Since any of these or other things may fail, it is not surprising that the duration of an average run without some failure is only a few hours. For example, a power failure (of which there were 9 in November and December, 1947) may spoil four or five tubes. Some of these failures are not clear-cut failures like a short, but borderline failures which may cause a tube to operate improperly once in a hundred to a thousand times. It is therefore important to be able to test the ENIAC rather thoroughly, rapidly and systematically, and, as far as possible, without touching any cables or switches. Testing of this kind is a science in itself and testing procedures will constantly be changed and improved. The procedure described in this section may accordingly be considered as only one stage in such a development. It is a program, set on the function tables which tests each unit an appropriate number of times and prints the errors found on IBM cards. The nature of the errors should localize the point of failure usually to within 10 or 20 tubes (if it is a tube failure) which can be replaced and examined at leisure without holding up the machine. Of course, the fact that part of the function table switches which describe the problem to be run must be changed to carry out the test is a defect, since switch errors or bad contacts may be made in setting the problem back. This defect is partially corrected by comparing the orders set on the function table with those set on cards before starting the problem. This latter test uses only a few lines of a function table.

**Function table test.** To test the function tables the number in the first row and first function table is sent to accumulator 11 or 15,  $10^n$  times, then shifted and subtracted from the correct number in the constant transmitter; if the difference is not zero, it is printed and then the next number is sent to accumulator 11 or 15, etc. The whole test takes about fifteen minutes if  $n = 3$ .

**Multiplier test.** A set of about 120 IBM cards is provided, each having two numbers,  $a_i$  and  $b_i$ , and the product  $c_i$ . The ENIAC reads a card, multiplies  $a_i$  by  $b_i$ ,  $10^n$  times, shifts the sum  $n$  places to the right, compares it with  $c_i$  and prints the difference if it is not zero; the next card is then read, etc. This test takes about a minute if  $n = 2$ .

**Divider and Square-rooter test.** Divisions and a square root are used.

**Accumulator test.** To test the accumulators, minus 0000000001 is sent from the constant transmitter to accumulator 1 whence it is sent to accumulator  $x$  and back to accumulator 15. Its sign is then changed and it is added to the number in accumulator 1; if the sum is not zero, it is printed. This is repeated  $m$  times, then the number is printed and then the value of  $x$  is changed. At present it is planned to test fourteen of the accumulators since the others are checked some other way. However, it may be desirable to check the others the same way. The testing time, if  $m$  is 1000, is about 5 minutes.

**Constant transmitter.** Two tests are provided for the constant transmitter. The first consists of sending minus 0000000001 from  $A_{LR}$   $m$  times, adding  $m$  and printing if not zero, then repeating for  $B_{LR}$ , etc. This only takes a few seconds. The second consists of reading the card 9999999999, -9999999999, 9999999999, -9999999999, etc., adding  $A_{LR}$  to  $B_{LR}$ ,  $C_{LR}$  to  $D_{LR}$ , etc., and printing if not zero. This is repeated fifty times, then the signs are changed, and it is repeated fifty times. The 9's are then reduced to

8's and the test proceeds. This tests the reader and the relay switches in the constant transmitter. About ten minutes are required for this test.

The whole program takes about thirty minutes for the values of m and n suggested, then probably another thirty minutes would be consumed by about four people to set and reset the 3600 switches required to run the program and then put the problem back on, and check the orders reset on the function tables.

Thus 1 to 1-1/2 hours will be required to run this functional test if all goes well. If a defective part is discovered, more time will be required to replace it and retest that unit (it is easy to start and stop the program at any two arguments in the function table).

A shorter, more elementary test which may be run every few hours to seek out major defects has been programmed. Since it only takes about 30 lines it may often be left on the function tables throughout the problem.





## SECTION VI

### FLOW DIAGRAM OF TYPICAL PROBLEM

Suppose that we wish to have the ENIAC compute the supersonic airflow past a body of revolution ABC, without yaw. If the Mach number  $M$  (velocity in units of sound velocity) is sufficiently large, the nose

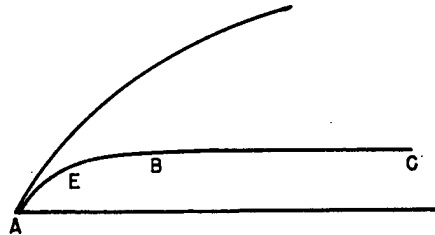


FIG. 6.1

angle sufficiently small on the curve ABC concave toward the axis, it is believed that there will exist an attached shock-wave front (surface of discontinuous pressure, density, and velocity) and the velocity will be everywhere supersonic. It is then reasonable to assume that replacing a small section AE of the nose contour AB by a straight line will have little influence on the flow at a distance from AE. This assumption does, however, have the following effects:

1. If the characteristic (curve making angle  $\omega = \csc^{-1} M$  with streamlines at each point, or curve of propagation of weak disturbances) from E, making an acute angle with the velocity vector, hits the shock wave at a point F, then the shock front is straight from A to F.

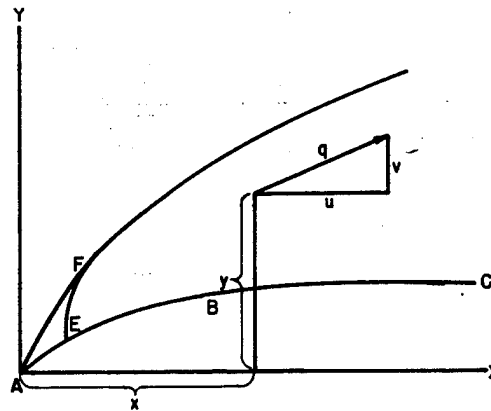


FIG. 6.2

2. In the region AEF, the partial differential equations defining velocity distribution can be satisfied by assuming that the velocity is constant on straight lines through A<sup>3</sup>. Thus  $u$  and  $v$  are functions of a single parameter in AEF and the partial differential equations become ordinary. It is the solution of these equations for  $x$ ,  $y$ ,  $u$ ,  $v$ , and  $z$  along EF which we shall choose as an illustrative problem.

We have then a system of ordinary equations for  $x$ ,  $y$ ,  $u$ ,  $v$  (see Figure 6.2) and  $z$  (stream function) in terms of some parameter along EF. We shall choose  $v/u$ , tangent of the inclination of the velocity vector, as this parameter.

<sup>3</sup>G.I. Taylor and J. W. Maccoll, "The Air Pressure on a Cone Moving at High Speeds", Proc. Roy. Soc. A139, 278-311 (1938).

For our purposes there is no need to derive the equations, which are:

$$\begin{aligned}
 6.1 \quad & \left\{ \begin{aligned}
 \frac{dx}{dt} &= (a^2 - u^2) F & q^2 &= u^2 + v^2 \\
 \frac{dy}{dt} &= (-uv - \sqrt{a^2(q^2 - a^2)}) F & a^2 &= \frac{\gamma-1}{2}(1 - q^2) \\
 \frac{du}{dt} &= -\frac{y u}{x + yt} & (1 - q^2)^{\frac{1}{\gamma-1}} &= A \\
 \frac{dz}{dt} &= yA(u \frac{dy}{dt} - v \frac{dx}{dt}) \\
 v &= ut \\
 F &= -\frac{y}{a^2 v} \frac{u}{(x + yt)} \frac{(a^2 - v^2)x + y(uv + \sqrt{a^2(q^2 - a^2)})}{uv + \sqrt{a^2(q^2 - a^2)}}
 \end{aligned} \right.
 \end{aligned}$$

$\gamma$ , the ratio of specific heats, is taken as 1.4 in the computations. There are initial and terminal conditions to be satisfied by  $x$ ,  $y$ ,  $u$ ,  $v$ ,  $z$ .

Clearly, at E

$$6.2 \quad \left\{ \begin{aligned} x &= x_E \\ y &= y_E \end{aligned} \right.$$

and  $z$  may be taken

$$6.3 \quad z = 0$$

Furthermore, the velocity vector must be parallel to AE:

$$6.4 \quad \left\{ \begin{array}{l} u = \frac{q_s}{\sqrt{1+t_s^2}} \\ v = \frac{q_s t_s}{\sqrt{1+t_s^2}} \end{array} \right. \quad \begin{array}{l} q_s \text{ is a parameter} \\ t_s = \text{slope of AE.} \end{array}$$

From the principles of conservation of mass, energy, and momentum follow the equations of Rankine and Hugoniot which must be satisfied at the shock-wave. Eliminating pressure and density from these equations we find the terminal condition which determines the value of  $t$  at the shock wave: namely, that if

$$6.5 \quad \phi(x, y, u, v) = (ux + vy) \left[ (y^2 + x^2) u - x(ux + vy) \right] \frac{\gamma + 1}{\gamma - 1} - x \left[ x^2 + y^2 - (ux + vy)^2 \right] \text{ and } u_w, v_w, x_w, y_w \text{ and } t_w$$

be the values of  $u, v, x, y$ , and  $t$  at the shock-wave, then  $\phi(u_w, v_w, x_w, y_w) = 0$ . Finally the velocity and Mach number in front of the shock-wave are determined by

$$6.6 \quad \left\{ \begin{array}{l} q_1 = \frac{u_w x_w + v_w y_w}{x_w} \\ M_1 = \sqrt{\frac{\frac{2}{\gamma-1} q_1^2}{1 - q_1^2}} \end{array} \right.$$

If  $\alpha$  and  $\beta$  are numbers, constant on characteristics,  $\alpha$  is some constant on EF and we shall take  $\beta$  arbitrarily as a linear function of  $t$

$$6.7 \quad \beta = B \frac{t - t_s}{t_w - t_s}$$

where  $B$  is some integer, for example, the number of cards printed.

The procedure is to assume some value for  $q_s$ ; integrate the equations with initial conditions by the method of Heun at equal steps in  $\Delta t$  up to the point where  $\phi$  changes sign; integrate backwards a shorter step  $\Delta t$  to the point determined by linear interpolation where  $\phi = 0$ . Let us call this complete process a "run".

Then  $q_1$  is computed, compared with its given value  $q_{1g}$ , and if  $|q_1 - q_{1g}| > \text{some given number } S$  the initial velocity  $q_s$  is adjusted by linear interpolation. New runs are then made (i.e., the process is reiterated) until  $|q_1 - q_{1g}| < S$ . Next  $\Delta t$  is taken equal to  $\frac{t_w - t_s}{n}$ , where  $n$  is some integer and the integration is repeated, printing the values of  $z, \alpha, \beta, v, u, -x, y$  every  $2^k$ th step until the shock-front is passed. One last step backwards is taken and the values of  $z, \alpha, \beta, v, u, -x, y$  on the shock-front are printed. Finally the two functions  $k_p$  and  $k_v$  of the velocity just behind the shock-front are computed and printed.

Before considering the flow diagram of this problem, let us consider the major detail in it, namely one step of Heun integration. Given a system of ordinary differential equations

$$6.8 \quad \frac{dp^h}{dt} = f(p^1, p^2, \dots, p^H, t) = f^h(p, t),$$

where  $h = 1, 2, \dots, H$

$$\text{and } p = [p^1, p^2, \dots, p^H],$$

and the values  $p_j^h$  of  $p^h$  at  $t = t_j$ ,

the procedure of Heun to find  $p_{j+1}^h = p^h(t_j + \Delta t)$

is to apply to the formulae:

$$6.9 \quad p_{j+1}^{*h} = p_j^h + \Delta t f^h(p_j, t_j)$$

$$6.10 \quad p_{j+1}^h = p_j^h + \Delta t \left[ f^h(p_j, t_j) + f^h(p_{j+1}^*, t_{j+1}) \right] / 2$$

Emphasizing the one step from  $t_j$  to  $t_{j+1}$ , let us introduce  $\delta t$  and a subscript  $1$  which is 1, 2, or 3 so that

$$6.11 \quad \left\{ \begin{array}{l} \delta t = \Delta t / 2 \\ p_j^h = (p_j^h)_0 \\ p_j^h + \delta t f^h(p_j, t_j) = (p_j^h)_1 \end{array} \right.$$

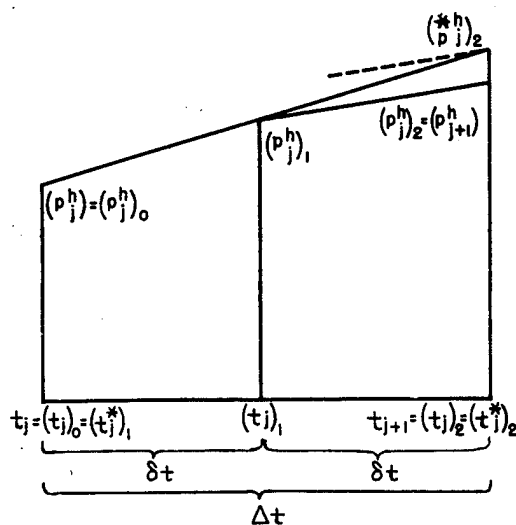


FIG. 6.3

$$6.11 \quad \left\{ \begin{array}{l} p_j^h + \Delta t f^h(p_j, t_j) = (p_j^h)_2 \\ \delta(p_j^h)_0 = 0 \\ (t_j^*)_1 = (t_j)_0 \end{array} \right.$$

$$(t_j^*)_2 = (t_j)_2 = (t_j)_0 + \Delta t$$

The following inductively defined sequences include equations 6.9 and 6.10:

$$6.12 \quad \left\{ \begin{array}{l} \delta(p_j^h)_1 = f^h \left[ (p_j^*)_1, (t_j^*)_1 \right] \delta t \\ (p_j^h)_1 = (p_j^h)_{1-1} + \delta(p_j^h)_{1-1} \\ (p_j^h)_1 = (p_j^h)_1 - \delta(p_j^h)_{1-1} + \delta(p_j^h)_1 \end{array} \right. \quad \begin{array}{l} p_j^h = (p_j^h)_0 \\ p_{j+1}^h = (p_j^h)_2 \end{array}$$

which must be evaluated for  $l = 1$  and  $l = 2$  to complete one step of Heun integration.

We may now begin to form a flow diagram of the problem. See von Neumann and Goldstine's report referred to in section 1. Essentially this problem is a triple induction; a Heun integration loop to be circled twice; a "run" loop to be repeated until  $\phi$  changes sign; and a "set of runs" loop to be repeated until the free stream velocity is satisfactory. We shall assign the induction (bound) variables  $l$ ,  $j$ , and  $i$  to these loops. The values of  $l$  are 1 and 2; of  $j$  are 1, 2, ...,  $J$  where  $J$  is the smallest integer such that  $\phi_j \geq 0$ ; of  $i$  are 1, 2, ...,  $I$  where  $I$  is the smallest integer such that

$$\left| (q_1)_i - (q_1)_g \right|^2 - s^2 \geq 0.$$

Actually, however, the problem is slightly complicated by the fact that there is always one last step and one last set of steps (which we shall call a last run) with a printing induction to which we assign the bound variable  $m$ . To distinguish between an ordinary step and a last step we shall store a quantity

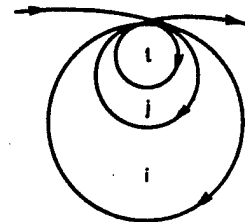


FIG. 6.4

$\epsilon_s = \begin{cases} +1 & \text{ordinary step} \\ -1 & \text{last step.} \end{cases}$  Similarly we shall use  $\epsilon_p = \begin{cases} 0, & \text{non-print run} \\ 2^k - m + 1, & \text{print run} \end{cases}$  to distinguish between a print run and a non-print run. With these definitions we may now develop the flow diagram as follows, showing in the alternative box at the exit to each loop, the variable whose change in sign switches the control out of the loop.

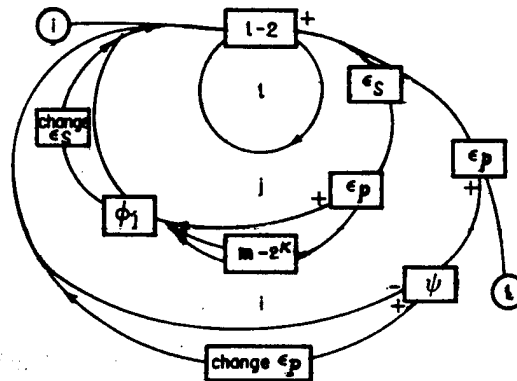


FIG. 6.5

Each of the decisions indicated by an alternative box is, of course, executed by a conditional transfer (order 75).

We are now ready to make a few decisions about the quantities we must store and then we can draw a more complete flow diagram showing what quantities are to be stored where in each operation box.

First. Since we plan to vary the initial conditions until we get the desired free stream velocity, we must save not only  $(q_s)_i$  but  $(q_s)_{i-1}$ . However, five figures will suffice so we may put them both in one accumulator, say D1. Furthermore both  $q_s$ 's are positive and therefore we may store  $\epsilon_s$  in D1 also.

Second. The other quantity we need in adjusting the initial conditions is  $(q_1)_{i-1}$ . This we may put in D2.

Third. Since we plan to take a last step backwards of magnitude  $\Delta t$  to be determined by interpolation with respect to  $\phi_j$  we must store  $\phi_j$  after each step. However we only need its sign until it changes sign at which time it is small; it will therefore suffice to store  $10^n \phi_j$  where  $n$  is suitably chosen. We shall store  $-10^3 \frac{\Delta t}{2}$  in the same accumulator which we may call D3.

Fourth. The quantities  $1-2$  and  $\epsilon_p$  being two or one digit numbers, we may put them in accumulator 6(10, 9) and 6(8, 7) respectively.

With these remarks about the bound variables, the following flow diagram (figure 6.6) will be clear. The boxes marked # are either substitution boxes which explain where the induction variables are changed or assertion boxes which state the conditions for leaving the alternative box by the unusual route.

A clear notion of the quantities held in each accumulator during each constancy interval (interval where no change in storage occurs) may be obtained from the storage table 6.I.

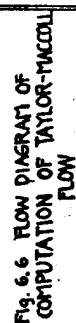




TABLE 8.1 STORAGE TABLE FOR COMPUTATION OF TAYLOR-MACCOLL FLOW

C.I.	Print ACC. 1 A1	Print ACC. 2 A2	ACC. 3 B1	ACC. 4 B2	ACC. 6 (6.5.4)	ACC. 6 (8.7)	ACC. 6 (11.10.9)	ACC. 7 A3	ACC. 8 P4	ACC. 9 A4	ACC. 10 A5	ACC. 11 C2	Print ACC. 15	Print ACC. 16 C1	Print ACC. 17 F3	Print ACC. 18 D1	Print ACC. 19 D2	Print ACC. 20 D3
.5	$K_p$	$K_p$	—	—	—	—	—	—	—	—	—	0	—	$\beta$	$t_s$	$q_1$	$M_1$	0
.6	$-x_A$	$q_g$	—	—	$\alpha$	—	—	—	—	—	—	—	—	—	$(q_s)_0$	$\epsilon_s (q_s)_{-1}$	$(q_1)_{-1} = 0$	$-\frac{4t}{2} \cdot 10^3$ (rt)
1.0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	$\epsilon_s (q_s)_{-1}$	$(q_1)_{-1}$	$-\frac{4t}{2} \cdot 10^3$ (rt)
2.0	$-x_s$	$y_s$	$\delta(-x_s)_{-1} = 0$	$\delta(y_s)_{-1} = 0$	—	$\epsilon_p$	—	$z = 0$	$\delta(u_s)_{-1} = 0$	$(u_s)_1$	$t_s$	—	—	—	$\delta(z_s)_{-1} = 0$	$(\epsilon_s, \beta + 1)$	—	$-\phi_j \cdot 10^3$ (left) and $-\frac{4t_w}{2} \cdot 10^3$ (rt)
3.0	—	—	—	—	Address for V	—	1-3 = -2	—	—	—	—	—	—	—	—	—	—	—
3.1	—	—	—	—	—	—	1-3	—	—	—	$(t_1)_{-1} + 4t$ $-(t_1)_{-1}$	—	—	—	—	—	—	—
4.0	$(-x_s)_1$	$(y_s)_1$	$\delta(-x_s)_1$	$\delta(y_s)_1$	—	—	1+1-3	$(z_s)_1$	$\delta(u_s)_1$	$(u_s)_1$	—	—	—	—	$\delta(z_s)_1$	—	—	—
4.05	$(-x_s)_{-1}$	$(y_s)_{-1}$	$\delta(-x_s)_{-1}$	$\delta(y_s)_{-1}$	—	—	1-3	$(z_s)_{-1}$	$\delta(u_s)_{-1}$	$(u_s)_{-1}$	—	—	—	—	$\delta(z_s)_{-1}$	—	—	—
4.1	—	—	—	—	—	—	1-3	—	—	—	—	—	—	—	—	—	—	—
4.2	$(-x_s)_2$	$(y_s)_2$	$\delta(-x_s)_2$	$\delta(y_s)_2$	—	—	0	$(z_s)_2$	$\delta(u_s)_2$	$(u_s)_2$	—	—	—	—	$\delta(z_s)_2$	—	—	—
5.0	—	—	—	—	Address for VI	—	—	—	—	—	—	—	$\epsilon_s$	$v_j$	—	—	—	—
5.1	—	—	—	—	—	—	—	—	—	—	—	—	$-\epsilon_p$	—	—	$\epsilon_s = +1$	—	—
6.0	—	—	—	—	Address for VII	—	—	—	—	—	—	—	—	—	—	—	—	—
6.1	—	—	—	—	—	$\epsilon_p$	—	—	—	—	—	—	—	—	—	—	—	—
6.2	—	—	—	—	Address for XVI	$2^{k-m-1}$	—	—	—	—	—	—	—	—	—	—	—	—
6.25	—	—	—	—	—	$2^k - m$	—	—	—	—	—	—	—	—	—	—	—	—
6.4	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—
6.15	—	—	—	—	—	0	—	—	—	—	—	—	—	—	—	—	—	—
6.3	—	—	—	—	—	$\epsilon_p$	—	—	—	—	—	—	—	—	—	—	—	—
7.0	—	—	—	—	Address for IX	—	—	—	—	—	—	—	$\phi_j \cdot 10^3$	$-\phi_j \cdot 10^3$	—	—	—	—
7.05	—	—	—	—	—	—	—	—	—	—	—	—	$\phi_{j-1} \cdot 10^3$	$-\phi_{j-1} \cdot 10^3$	—	—	—	—
7.1	—	—	—	—	—	—	—	—	—	—	—	—	—	$-\phi_j \cdot 10^3$	—	—	—	—
7.2	—	—	—	—	—	—	—	—	—	—	—	—	—	$-\phi_j \cdot 10^3$	—	—	—	—
7.2	—	—	—	—	—	—	—	—	—	—	—	—	—	$-\phi_j \cdot 10^3$	—	—	—	—
5.2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	$\epsilon_s = -1$	—	—
8.0	—	—	—	—	Address for XI	$\epsilon_p = 0$	—	$(q_1)_1$	—	—	—	—	$-\epsilon_p$	—	—	—	—	—
8.1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
9.0	—	—	—	—	Address for XIII	—	—	—	—	—	—	—	$S^2 - [(q_1)_1 - (q_1)_g]^2$	—	—	—	—	—
9.05	—	—	—	—	—	—	—	$(q_1)_{1-1}$	—	—	—	—	$S^2 - [(q_1)_{1-1} - (q_1)_g]^2$	—	—	—	—	—
9.1	—	—	—	—	—	—	—	$(q_1)_{1-1}$	—	—	—	—	$S^2 - [(q_1)_{1-1} - (q_1)_g]^2$	—	—	—	—	—
9.2	—	—	—	—	—	—	—	$(q_1)_{1-1}$	—	—	—	—	$S^2 - [(q_1)_{1-1} - (q_1)_g]^2$	—	—	—	—	—
8.2	—	—	—	—	$\epsilon_p = 1$	—	—	$(q_1)_1$	—	—	—	—	$S^2 - [(q_1)_1 - (q_1)_g]^2$	—	—	—	—	—

## SECTION VII

### CODING THE OPERATION BOXES

It is now a simple matter to consider the operation boxes, one at a time, and write the succession of orders which will cause the machine to carry out the computations described in that operation box. For the convenience of the reader we have written these orders both by name and by number for one example. (See Tables 7.I and 7.II) The reader can learn the code by making a table twenty columns wide and translating the code numbers into the movement of numbers among these columns.

Finally, it is advisable to prepare a test run by giving all the variables numerical values and carrying out one typical computation for testing occasionally that the Eniac is operating properly. Such a test run has been computed for this problem but it does not seem necessary to include it in this report.

## SECTION VIII

### MODIFIED ENIAC

Two additions to the ENIAC have been ordered which will greatly increase its memory and flexibility. The first is called the converter. It is a device for converting any two-digit number to a program pulse in one add-time. This will make available a hundred pulses for orders instead of sixty and will, in addition, free the master programmer for other uses. The converter has already been delivered and has already been used to modify the sixty-order code in such a way as to save space in the function tables for shifts (i.e. there are 19 two-digit shift orders instead of one four-digit shift order) and to make it possible to take orders from cards as well as from the function tables.

We shall not describe this modified code in detail since there will be a better code available by the time this report is available. The better code will use not only the converter but the other addition called the register.

The register is really a set of one hundred ten-digit registers with three operations: it can clear its old argument and receive a new argument in one add-time; it can receive a number, clear that register designated by its current argument and send the number to that register, increasing its argument by one; it can send out (and also keep) the number designated by its current argument. The latter two operations consume three add-times.

With the converter and register it will be possible to achieve among others the following results which will be incorporated in a new code to be described when completed:

1. Increase the speed of the ENIAC considerably
2. Increase the efficiency of the orders; that is, the amount of computation per unit space in the function tables.
3. Store orders in the register and thereby make it possible to control computations with cards and to modify orders for inductive processes.
4. Use the additional memory to solve problems involving more numbers, e.g., solve a system of thirty ordinary non-linear differential equations.
5. Use the card control to facilitate testing and demonstrating the ENIAC.

A first form of this new code should soon be available and the ENIAC should be ready to use this code by December.





1 Mar 1948

TABLE 7.11 INPUT DATA - AXIAL FLOW - NEW 80 ORDER CODE

F.T. I										F.T. II										F.T. III														
L	00	62	80	02	44	01	83	00	31	33	01	32	34	02	00	01	95	20	92	02	74	00	Initial Pulse					CT on $-\epsilon_p$ CT on $-(\epsilon_p - 1)$						
SEQ 0	01	24	44	25	91	82	24	01	40	53	10	42	41	12	02	93	03	02	34	74	00	00												
	02	44	23	81	22	44	61	02	42	14	43	30	90	01	03	02	93	03	02	34	74	00	00											
	03	00	94	04	86	11	72	03	22	44	30	63	21	52	04	14	05	04	70	01	83	93	83	75										
	04	90	95	24	91	53	24	04	14	52	30	63	22	52	05	51	05	94	02	09	73	00	00											
L	05	25	95	01	80	09	74	06	85	94	45	30	14	51	07	08	09	95	22	82	10	74	00											
	06							SEQ 07	44	21	51	52	63	30	08																			
	07							08	60	90	91	05	00	52	09																			
	08							09	44	32	43	30	14	42	10																			
F	09	82	13	54	90	05	12	III	10	30	20	40	14	50	10																			
SEQ I	10	43	14	70	01	85	92	11	35	15	51	90	92	55	11																			
	11	45	20	43	30	80	90	12	21	44	06	42	14	45	Heun																			
	12	91	10	42	55	12	83	13	12	43	30	30	42	63																				
	13	02	44	01	03	04	10	14	15	44	12	35	14	45																				
L	14	11	23	93	63	93	75	15	90	91	55	20	45	10																				
	15							16	50	15	52	55	22	44																				
	16							17	10	80	00	31	63	44																				
	17							18	90	91	20	32	14	45																				
F	18	70	02	63	92	00	95	19	05	40	02	43	30	10																				
SEQ II	19	02	01	00	74	00	00	20	42	15	35	12	45	30																				
F	20	42	14	43	30	90	01	21	55	05	32	10	44	02																				
SEQ	21	22	95	20	10	22	74	22	80	00	44	63	31	14																				
V	22	54	75	00	40	21	80	23	52	20	45	22	50	30																				
F	23	00	31	63	44	90	91	24	14	32	90	91	44	14																				
X	24	10	52	14	32	30	55	25	35	30	90	91	14	51																				
	25	90	01	42	10	84	11	26	30	90	01	63	21	42																				
	26	80	14	95	23	70	20	27	30	90	01	12	52	14																				
	27	74	00	00	00	00	00	28	43	30	90	01	14	44																				
F	28	93	63	75	00	52	23	29	30	20	52	14	52	30																				
XVII	29	70	10	90	05	63	54	30	63	65	14	35	15	44																				
	30	22	32	90	91	24	91	31	90	92	60	90	01	20																				
	31	31	90	91	25	42	24	32	45	05	50	30	30	14																				
	32	51	90	01	01	02	81	33	32	30	15	52	14	45																				
	33	00	44	61	00	53	22	34	22	43	30	90	01	15																				
	34	40	03	40	14	40	30	35	44	20	51	14	50	21																				
	35	15	45	63	65	90	91	36	45	30	20	51	15	44																				
	36	10	94	04	87	11	72	37	21	45	14	42	30	20																				
	37	90	94	44	14	45	30	38	52	15	44	22	60	14																				
	38	90	92	55	60	04	52	39	45	30	90	01	15	91																				
	39	14	52	30	21	40	90	40	54	90	05	90	91	83																				
	40	01	14	44	90	92	80	41	14	91	45	30	90	01																				
	41	90	01	30	30	11	41																											

Compute  $-4tw/2$ CT on  $\epsilon^2 - (q_w - q_p)^2$

TABLE 7.11 (Continued)

F.T. I										F.T. II										F.T. III												
42	30	30	05	24	91	33				42	15	53	63	40	45	10				42	54	15	80	14	40	11						
43	24	34	25	44	03	70				43	45	23	42	30	90	01				43	41	63	64	90	91	10						
44	35	85	94	45	14	51				44	15	34	63	32	45	02				XII	44	44	63	41	55	14	45					
45	30	15	33	90	94	63				III	45	45	04	51	30	90	01				45	30	90	01	54	90	95					
46	45	04	70	60	85	92				46	15	33	63	31	45	01				46	54	24	41	25	91	94						
47	45	14	35	30	90	91				47	45	03	35	63	30	90				47	04	86	11	72	90	95						
48	10	34	55	01	51	33				48	91	15	41	63	52	45					48	24	91	95	01	80	09					
49	14	41	30	90	91	10				49	12	45	11	70	01	92				L	49	74	00	00	00	00	00					
50	70	60	85	94	45	14				L	50	75	00	44	44	90	93				50											
51	51	30	55	02	82	23				IV	51	43	13	94	01	00	73				I'	51	91	82	63	43	13	70				
52	81	22	44	61	00	71				52										XIII	52	2 <sup>k+1</sup>	85	92	45	14	94					
53										53											53	12	80	85	94	45	30					
54										54											54	10	43	55	90	94	83					
55	81	22	44	21	42	14				55											55	24	91	54	65	24	95					
56	43	30	90	01	23	91				56											L	56	05	50	09	74	00	00				
57	54	11	32	90	91	24	First			57																						
58	91	42	24	31	90	91	Print			58																						
59	25	01	02	51	61	00				59																						
60	41	24	91	83	02	44				I	60	54	11	32	90	91	24															
61	01	70	10	90	05	52				SEQ	61	91	54	05	31	90	91															
62	24	70	2 <sup>k</sup>	93	00	11				XVI	62	25	42	24	52	23	41															
63	23	94	00	18	73	00				63	22	40	90	91	01	02																
64										64	81	00	44	61	21	70																
65										65	10	90	05	52	21	35																
66										66	24	91	54	90	01	02																
67										67	53	22	84	90	01	01																
68										68	51	24	70	2 <sup>k</sup>	93	00																
69										L	69	94	02	09	73	00	00															
70										70																						
71																																
72																																
73																																
74																																
75																																
76																																
77																																
78											90	00	00	00	xx	xx	xx															
79										91																						
80										92																						
81																																
82																																
83																																

$-\Delta t/2 \times 10^3 \beta$  side (send in arg. 86)  
 $2/(\gamma - 1)$   
 $(\gamma + 1)/(\gamma - 1)$

$$-4\sqrt{2} \times 10^3 \beta \text{ slide (send in arg. 86)}$$

$$2/(\gamma - 1)$$

$$(\gamma + 1)/(\gamma - 1)$$

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